$$abc + \bar{b}\bar{c}d + b\bar{c} + \bar{b}ce + b\bar{c}f =$$

$$= abc + \bar{b}\bar{c}d + b\bar{c}(\cancel{1+f}) + \bar{b}ce$$

$$= abc + \bar{b}\bar{c}d + b\bar{c} + \bar{b}ce =$$

$$= b(ac + \bar{c}) + \bar{b}\bar{c}d + \bar{b}ce =$$

$$= b(\bar{c} + a) + \bar{b}\bar{c}d + \bar{b}ce =$$

$$= b\bar{c} + ba + \bar{b}\bar{c}d + \bar{b}ce =$$

$$= \bar{c}(b + \bar{b}d) + ab + \bar{b}ce =$$

$$= \bar{c}(b + d) + ab + \bar{b}ce =$$

$$= b\bar{c} + \bar{c}d + ab + \bar{b}ce.$$

$$x + \bar{x}y = x + y$$

$$(a+c)(\bar{a}+\underset{x}{b}+d)(\underset{x}{b}+c+\bar{d})=$$

x y x z

$$(x+y)(x+z)=x+yz$$

$$x+\bar{x}y=x+y$$

$$=(a+c)\left(b+(\bar{a}+d)\cdot(c+\bar{d})\right)=$$

$$=(a+c)(b+\bar{a}c+\bar{a}\bar{d}+cd+\cancel{d\bar{d}})=$$

$$=ab+\cancel{a\bar{a}c}+\cancel{a\bar{a}\bar{d}}+acd+bc+\bar{a}c\cancel{c}+\bar{a}c\bar{d}+c\cancel{c}d=$$

$$=ab+acd+bc+\bar{a}c+\bar{a}c\bar{d}+cd=$$

$$=ab+c(ad+\bar{a})+bc+c(\bar{a}\bar{d}+d)=$$

$$=ab+c(\bar{a}+d)+bc+\cancel{c(d+\bar{a})}=$$

$$=ab+\bar{a}c+cd+bc$$

$(\underset{y}{a}+\underset{x}{c})\cdot(\bar{a}+b+d)(\underbrace{b+\underset{x}{c}+\bar{d}}_{z})$ $\qquad$ $(x+y)(x+z)=x+yz$

$= (c+a(b+\bar{d}))\cdot(\bar{a}+b+d) =$

$= c\bar{a}+c(b+d)+\cancel{a(b+\bar{d})\bar{a}}+a(b+\bar{d})(b+d) =$

$= \bar{a}c+c(b+d)+ab =$

$= \bar{a}c+bc+cd+ab$

$$(a+c)(\bar{a}+b+d)(b+c+\bar{d})=$$

$$=(\cancel{a\bar{a}}+ab+ad+\bar{a}c+bc+cd)\cdot(b+c+\bar{d})=$$

$$=ab\cancel{b}+abc+ab\bar{d}+adb+adc+\cancel{ad\bar{d}}+\bar{a}cb+\bar{a}c\cancel{c}+\bar{a}c\bar{d}+$$

$$+bc\cancel{b}+bc\cancel{c}+bc\bar{d}+cdb+cd\cancel{c}+\cancel{cd\bar{d}}=$$

$$=ab+abc+ab\bar{d}+abd+adc+\bar{a}cb+\bar{a}c+\bar{a}c\bar{d}+$$

$$+bc+bc+bc\bar{d}+cdb+cd=$$

$$=ab(\cancel{1+c+\bar{d}+d})+cd(\cancel{a+1})+\bar{a}c(\cancel{b+1+\bar{d}})+bc(\cancel{1+1+\bar{d}+d})=$$

$$=ab+cd+bc+\bar{a}c$$

$$a\bar{b}c + cdf + d\bar{e} + e\bar{f} + \overline{bd} =$$

$$= a\bar{b}c + cdf + d\bar{e} + e\bar{f} + \bar{b} + \bar{d} =$$

$$x + \bar{x}y = x + y$$

$$= \bar{b}(\cancel{ac+1}) + \bar{d} + \bar{e} + ef + cdf =$$

$$= \bar{b} + \bar{d} + cf + \bar{e} + \bar{f} =$$

$$= \bar{b} + \bar{d} + \bar{e} + \bar{f} + c$$

$x\oplus y = x\bar y + \bar x y$

$$(a+(b\oplus\bar c))\cdot\overline{(c+(a\oplus\bar b))}=$$

$$=(a+bc+\bar b\bar c)\cdot\bar c\cdot\overline{a\oplus\bar b}$$

$$=(a\bar c+\cancel{bc\bar c}+\bar b\bar c\cancel{\bar c})\cdot\overline{ab+\bar a\bar b}=$$

$$=(a\bar c+\bar b\bar c)\cdot\overline{ab}\cdot\overline{\bar a\bar b}=$$

$$=(a\bar c+\bar b\bar c)\cdot(\bar a+\bar b)\cdot(a+b)=$$

$$=(a\bar c+\bar b\bar c)\cdot(\cancel{\bar a a}+\bar a b+a\bar b+\cancel{b\bar b})=$$

$$=\cancel{a\bar c\bar a b}+a\bar c\cancel{a}\bar b+\cancel{\bar b\bar c\bar a b}+\bar b\bar c a\cancel{\bar b}=$$

$$=a\bar b\bar c+a\bar b\bar c$$

$$=a\bar b\bar c$$

$$\left(a+(b\oplus\bar c)\right)\cdot\overline{\left(c+(a\oplus\bar b)\right)} = \qquad\qquad x\oplus y = x\bar y + \bar x y$$

$$= (a+bc+\bar b\bar c)\cdot\overline{c+ab+\bar a\bar b} =$$

$$= (a+bc+\bar b\bar c)\cdot\bar c\cdot\overline{ab}\cdot\overline{\bar a\bar b} =$$

$$= (a+bc+\bar b\bar c)\cdot\bar c\cdot(\bar a+\bar b)\cdot(a+b) =$$

$$= a\cdot\bar c(\bar a+\bar b)(a+b) + \cancel{bc\bar c(\ )(\ )} + \bar b\bar c\bar c(\bar a+\bar b)(a+b) =$$

$$= \cancel{a\bar c\bar a(a+b)} + a\bar c\bar b(a+b) + ab\bar c(\bar a+\bar b) + \cancel{b\bar b(\bar a+\bar b)} =$$

$$= a\bar b\bar c + \cancel{a\bar c\bar b b} + \cancel{a\bar b\bar c\bar a} + a\bar b\bar c =$$

$$= a\bar b\bar c$$

$$(ab)\oplus c + (\bar b\cdot c)\oplus \bar a + \overline{b\oplus c} =$$

$$= \overline{ab}\cdot c + ab\cdot\bar c + \overline{\bar b c}\cdot\bar a + \bar b c\cdot a + \overline{b\bar c + \bar b c} =$$

$$= (\bar a+\bar b)\cdot c + ab\bar c + (b+\bar c)\cdot\bar a + a\bar b c + \overline{b\bar c}\cdot\overline{\bar b c} =$$

$$= \bar a c + \bar b c + ab\bar c + \bar a b + \bar a\bar c + a\bar b c + (\bar b + c)(b+\bar c) =$$

$$= \bar a c + \bar b c + ab\bar c + \bar a b + \bar a\bar c + a\bar b c + \cancel{\bar b b} + \bar b\bar c + bc + \cancel{c\bar c}$$

$$x + \bar x y = x + y$$

$$= \bar a(\cancel{c+\bar c}) + \bar b c(\cancel{1+a}) + b(a\bar c+\bar a) + \bar b\bar c + bc =$$

$$= \bar a + \bar b c + b(\bar a+\bar c) + \bar b\bar c + bc =$$

$$= \bar a + \bar b(\cancel{c+\bar c}) + b(\cancel{\bar a+\bar c+c}) =$$

$$= \bar a + \cancel{\bar b + b} = a + 1 = 1$$